

Soft Multiset: A Different View

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Abstract

The concept of soft multiset theory was introduced in 2011 but the condition on which it was defined does not actually convey the notion of multiset. Babitha and Sunil in 2012 captured the concept of multiset in their definition of soft multiset but with restriction on the mapping of set of parameters. In this paper, the concept of soft multiset is defined without restriction and presents its various operations with their properties. Some important results are also established.

Keywords: Multiset, soft set, soft multiset, multi-value class, operations.

1. Introduction

Most of the problems encountered in engineering, medical sciences, economics, environments, social sciences, etc., have various uncertain attributes. Molodtsov [1] study the concept of soft set theory as a general mathematical tool for dealing with such uncertainties. The origin of soft set theory could be traced to the work of Pawlak [2] in 1993 titled *Hard and soft set* in proceeding the international EWorkshop on rough sets and knowledge discovery at Banff. His notion of soft sets is a unified view of classical rough and fuzzy sets. This might have motivated D. Molodtsov's work in 1999 titled *soft set theory: first result*. There in, the basic notions of the theory of soft sets and some of its possible applications were presented. For positive motivation, the work discusses some problems of the future with regards to the theory. This theory to some extent is free from the inadequacy of the parameterization tools of other nonstandard set theory.

Soft set could be regarded as neighbourhood systems, and they are special case of context-dependent fuzzy sets. In soft set theory the problem of setting the membership function in fuzzy set, among other related problems does not arise. This makes the theory very convenient and easy to apply in practice as in [3, 4, 5].

After Molodtsov's work, different operations [6, 7, 1, 8, 9] and applications of soft sets [4, 10] and derivatives of soft set [11, 12, 13] were studied.

In this paper, the concept of soft multiset is redefined to capture the concept of multiset. Various operations are also redefined in the light of the new concept. Properties of the operations are also investigated and some results established.

2. Preliminaries

Definition 2.1 Soft set [1]

Let U be a universal set and let $A \subseteq E$ be a set of parameters (each parameter could be a word or a sentence). Let $P(U)$ denotes the power set of U . A pair (F, A) is called a *soft set* over a given universal set U , if and only if F is a mapping of a set of parameters E , in to the power set of U . That is, $F: A \rightarrow P(U)$.

A soft set over U is a parameterized family of subsets of a given universe U . Also, for any $e \in A$, $F(e)$ is considered as the set of e –approximate elements of the soft set (F, A) .

Example 2.1

- (i) Let (X, τ) be a topological space, that is, X is a set and τ is a topology (a family of subsets of X). Then, the family of open neighbourhoods $T(x)$ of points x , where $T(x) = \{V \in \tau: x \in V\}$, may be considered as the soft set $(T(x), \tau)$.
- (ii) Let A be a fuzzy set and λ_A be a membership function of the fuzzy set A , that is, λ_A is a mapping of U into $[0, 1]$, let $F(\mathcal{C}) = \{x \in U: \lambda_A(x) \geq \mathcal{C}\}$, $\mathcal{C} \in [0, 1]$ be a family of \mathcal{C} -level sets for function λ_A . If the family F is known, $\lambda_A(x)$ can be founded by means of the definition: $\lambda_A(x) = \text{Sup}_{\mathcal{C} \in [0,1]} \mathcal{C}$. Hence every fuzzy set A may be considered as the soft set $(F, [0, 1])$.
- (iii) Let $U = \{C_1, C_2, C_3, C_4, C_5, C_6, C_7, C_8, C_9, C_{10}\}$ be set of cars under consideration, E be a set of parameters.
 $E = \{e_1 = \text{expensive}, e_2 = \text{beautiful}, e_3 = \text{manual gear}, e_4 = \text{cheap}, e_5 = \text{automatic gear}, e_6 = \text{in good repair}, e_7 = \text{in bad repair}\}$. Then the soft set (F, A) describes the attractiveness of the cars.

Definition 2.2 Soft Multiset [13]

Let $\{U_i: i \in I\}$ be a collection of universes such that $\cap U_i = \emptyset$ and let $\{E_{U_i}: i \in I\}$ be a collection of sets of parameters. Let $U = \prod_{i \in I} P(U_i)$, where $P(U_i)$ denotes the power set of U_i , $E = \prod_{i \in I} E_{U_i}$ and $A \subseteq E$. A pair (F, A) is called a soft multiset over U , where F is a mapping given by $F: A \rightarrow U$.

In other words, a soft multiset over U is a parameterized family of subsets of U . For $\varepsilon \in A, F(\varepsilon)$ may be considered as the set of ε -approximate elements of the soft multiset (F, A) .

Remark 1

In [9] if $\cap U_i = \emptyset$, then $\cup U_i$ is just an ordinary set, hence the concept of multiset does not exist. Hence, the remaining exercise collapses to soft set.

Definition 2.3. [11] Let U be universal multiset and E be set of parameters. Then an ordered pair (F, E) is called a soft multiset where F is a mapping given by $F: E \rightarrow PW(U)$

Remark 2

In [11], because of the restriction on the submultisets, some of the submultisets are ignored hence the definition does not completely capture the notion of soft set.

2.3 Concept of multiset [14]

A multiset (Mset, for short) is an unordered collection of objects (called the elements) in which, unlike a standard (Cantorian) set, elements or objects are allowed to repeat. In other words an Mset A over D is a pair (D, F) such that $F: D \rightarrow N$ is a function where N is a set of natural number, $\{0, 1, 2, \dots, n, \dots\}$. D is called the ground set or the domain set of all the multiset constructed from D . The number of times an element x occurs in A is called the multiplicity of that element denoted by $M_A(x)$ or $A(x)$ multiset is generally abbreviated as mset.

2.3.1 Some Basic Definitions in Multiset

- a. The mset A for any ground set D is called empty, denoted by \emptyset or $[\]$, if $M_A(x) = 0$ for any $x \in D$.
- b. For a given mset A , the set of all its objects is called its root set usually denoted by A^* , and the sum of the multiplicities of all its objects is called its cardinality, usually denoted by $C(A)$ or $|A|$.
- c. An mset is called 'regular' or 'constant' if all its elements occur with the same multiplicity. Also an mset is called 'simple' if all its elements are the same, for instance, $[x]_4$ is a simple mset containing x as its only object.
- d. Two msets A and B are said to be equal, written as $A=B$, if and only if for any object $x \in D$, $M_A(x) = M_B(x)$ or $A(x) = B(x)$. Equivalently, $A=B$ if every element of A is in B and conversely. Clearly, $A = B \Rightarrow A^* = B^*$, however the converse need not hold.
- e. Two msets A and B are said to be 'cognate' or similar if $\forall x (x \in A \Leftrightarrow x \in B)$, where x is an object. Thus, similar msets have equal root sets but need not be equal, themselves.
- f. Let A and B be two msets, A is an msubset or a submultiset of B , written as $A \subseteq B$ or $B \supseteq A$ and $A \neq B$, then A is called a proper submultiset of B . An mset is called parent in relation to its msubsets.

It is easy to show that \subseteq is antisymmetric, that is $A \subseteq B$ and $B \subseteq A \Rightarrow A = B$, and it is a partial ordering on the class of msets defined on a given generic domain. Clearly, \emptyset is a subset of every set.

- g. Relatedly, a ‘whole’ msubset of a given mset contains all multiplicities of common elements; while a ‘full’ msubset contains all objects of the parent mset and according, every mset contains a unique full msubset, called its root set. Clearly, for any two msets A and B if $A \subseteq B$ and $Dom(A) = Dom(B)$, then A is a full msubset of B .
- h. The power of multiset of a given mset A , denoted by $\tilde{P}(A)$, to distinguish it from the symbol $P(A)$ used for power set of A , is the multiset of all submultisets of A .
- i. Let A and B be two mset over a given domain set D . $A \cup B$ is the mset defined by $M_{A \cup B}(x) = M_A(x) \cup M_B(x) = \text{Maximum}(M_A(x), M_B(x))$, being the union of two numbers.
- j. Let A and B be two mset over a given domain set D . $A \cap B$ is the mset defined by $M_{A \cap B}(x) = M_A(x) \cap M_B(x) = \text{Minimum}(M_A(x), M_B(x))$, being the intersection of two numbers. That is, an object x occurring a times in A and b times in B , occurs minimum (a,b) times in $A \cap B$, which always exists.
- k. Let A and B be two mset over a given domain set D . $A + B$ or $A \uplus B$ is the mset defined by $M_{A+B}(x) = M_A(x) + M_B(x)$ for any $x \in D$, direct sum of two numbers. That is, an object x occurring a times in A and b times in B , occurs $a + b$ times in $A \uplus B$.

3. Redefined Soft multiset theory

Definition 3.1 Let $\{U_i : i \in I\}$ be a collection of universes such that there exist U_j, U_k and $U_j \cap U_k \neq \emptyset$. Suppose $U = \uplus_{i \in I} P(U_i)$, where $P(U_i)$ denotes the power set of U_i , and E be a set of parameters. A pair (F, A) , where $A \subseteq E$, is called a soft multiset over U . F is a mapping given by $F: A \rightarrow U$. That is, a soft multiset over U is a parametrized family of submultisets of U such that for $e \in A$, $F(e)$ is considered as the set of e -approximate element of the soft multiset (F, A) .

Examples 3.1

Let $\mathcal{S}_i : i \in \mathcal{N}$ be a collection of states in a country and $U_i : i \in \mathcal{N}$ be a collection of states with availability of land, labour and raw materials.

Suppose

$U_1 = \{\mathcal{S}_1, \mathcal{S}_2, \mathcal{S}_3\}$ be a set of states with availability of land,

$U_2 = \{\mathcal{S}_2, \mathcal{S}_4, \mathcal{S}_6\}$ be a set of states with availability of labour,

$U_3 = \{\mathcal{S}_2, \mathcal{S}_4, \mathcal{S}_7\}$ be a set of state with availability of raw materials.

$$\begin{aligned}
 U &= P(U_1) \uplus P(U_2) \uplus P(U_3) \\
 &= \left\{ \{\mathcal{S}_1\}, \{\mathcal{S}_2\}, \{\mathcal{S}_3\}, \{\mathcal{S}_1, \mathcal{S}_2\}, \{\mathcal{S}_1, \mathcal{S}_3\}, \{\mathcal{S}_2, \mathcal{S}_3\}, \{\mathcal{S}_1, \mathcal{S}_2, \mathcal{S}_3\}, \emptyset, \{\mathcal{S}_2\}, \{\mathcal{S}_4\}, \{\mathcal{S}_6\}, \{\mathcal{S}_2, \mathcal{S}_4\}, \{\mathcal{S}_2, \mathcal{S}_6\}, \{\mathcal{S}_4, \mathcal{S}_6\}, \right. \\
 &\quad \left. \{\mathcal{S}_2, \mathcal{S}_4, \mathcal{S}_6\}, \emptyset, \{\mathcal{S}_2\}, \{\mathcal{S}_4\}, \{\mathcal{S}_7\}, \{\mathcal{S}_2, \mathcal{S}_4\}, \{\mathcal{S}_2, \mathcal{S}_7\}, \{\mathcal{S}_4, \mathcal{S}_7\}, \{\mathcal{S}_2, \mathcal{S}_4, \mathcal{S}_7\}, \emptyset, \right\} \\
 &= \\
 &\left\{ \{\mathcal{S}_1\}, 3\{\mathcal{S}_2\}, \{\mathcal{S}_3\}, 2\{\mathcal{S}_4\}, 3\emptyset, \{\mathcal{S}_6\}, \{\mathcal{S}_7\}, \{\mathcal{S}_1, \mathcal{S}_2\}, \{\mathcal{S}_1, \mathcal{S}_3\}, \{\mathcal{S}_2, \mathcal{S}_3\}, 2\{\mathcal{S}_2, \mathcal{S}_4\}, \{\mathcal{S}_1, \mathcal{S}_2, \mathcal{S}_3\}, \right. \\
 &\quad \left. \{\mathcal{S}_2, \mathcal{S}_6\}, \{\mathcal{S}_4, \mathcal{S}_6\}, \right. \\
 &\quad \left. \{\mathcal{S}_2, \mathcal{S}_4, \mathcal{S}_6\}, \{\mathcal{S}_2, \mathcal{S}_7\}, \{\mathcal{S}_4, \mathcal{S}_7\}, \{\mathcal{S}_2, \mathcal{S}_4, \mathcal{S}_7\} \right\}
 \end{aligned}$$

this is a multiset.

Let \mathcal{E} be a set of decision parameters related to the above universes, where

$\mathcal{E} = \{e_1 = \text{peaceful}, e_2 = \text{kidnapping}, e_3 = \text{Armed robbery}, e_4 = \text{Accessibility}, e_5 = \text{market}\}$.

Let $\mathcal{A} = \{e_1 = \text{peaceful}, e_2 = \text{kidnapping}, e_3 = \text{Armed robbery}, e_4 = \text{Accessibility}\}$.

The Soft multiset $(\mathcal{F}, \mathcal{A})$ is a parametrized family $\{\mathcal{F}(e_i), i = 1, 2, \dots, 4\}$ of subsets of the set U and gives us a collection of approximate description of the conditions of some states in a country favourable to Mr. \mathcal{X} for the location of his manufacturing industries.

Definition 3.2: Multivalued-class.

The class of all value set of a soft multiset (F, A) is called the value class of the soft multiset and is denoted by

$$C_{(F,A)}^* = \{V_1, V_2, \dots, V_n\}. \text{ Obviously}$$

$$C^*(F, A) \subseteq U. \text{ Also, if there exists at least one } i \text{ such that } V_i = V_j,$$

$\forall i, j = 1, 2, \dots, n$, then the value-class of the soft multiset (F, A) is called Multivalue-class of the soft multiset (F, A) and is denoted by $C_{(F,A)}^m$. Similarly, $C_{(F,A)}^m \subseteq U$.

Definition 3.3: Soft multisubset.

Let (F, A) and (G, B) be two softmultisets over U , we say that (F, A) is a soft multisubset of (G, B) written as $(F, A) \tilde{\subseteq} (G, B)$ if

- i. $A \subseteq B$
- ii. $\mathcal{M}((F, A))(x) \leq \mathcal{M}((G, B))(x)$ for all $x \in U$.

Definition 3.4: Equality of two soft multisets

Two Soft multisets (F, A) and (G, B) over U are said to be equal if and only if (F, A) is a soft multisubset of (G, B) and (G, B) is a soft multisubset of (F, A) .

Definition 3.5: NOT Set of a set parameters.

Let E be a set of parameters. The NOT set of E denoted by $\lrcorner E$ is defined by $\lrcorner E = \{\lrcorner e_1, \lrcorner e_2, \dots, \lrcorner e_n\}$ where $\lrcorner e_i = \text{not } e_i, \forall i$.

Proposition 3.1

- 1. $\lrcorner(\lrcorner A) = A$
- 2. $\lrcorner(A \cup B) = (\lrcorner A \cup \lrcorner B)$
- 3. $\lrcorner(A \cap B) = (\lrcorner A \cap \lrcorner B)$

Definition 3.6: Similar Soft multisets

Two Soft multisets (F, A) and (G, B) are said to be ‘Cognate’ or similar if $\forall x (x \in (F, A) \Leftrightarrow x \in (G, B))$ where x is an object. Therefore, similar Softmultisets have equal

root sets but need not be equal themselves.

Definition 3.7: Union of two soft multisets

Let (F, A) and (G, B) be two Softmultisets over U . $(F, A) \cup (G, B)$ is the soft multiset defined by

$$M_{(F,A) \cup (G,B)}(x) = M_{(F,A)}(x) \cup M_{(G,B)}(x) = \text{maximum} \left(M_{(F,A)}(x), M_{(G,B)}(x) \right)$$

being the union of two numbers.

Definition 3.8: Intersection of two soft multisets

Let (F, A) and (G, B) be two soft multisets over U . Then, the intersection of (F, A) and (G, B) written as $(F, A) \cap (G, B)$ is the Soft multiset defined by

$$M_{(F,A) \cap (G,B)}(x) = M_{(F,A)}(x) \cap M_{(G,B)}(x) = \text{minimum} \left(M_{(F,A)}(x), M_{(G,B)}(x) \right)$$

being the intersection of two numbers. That is, an object x occurring a times in (F, A) and b times in (G, B) , occurs minimum (a, b) in $(F, A) \cap (G, B)$, which always exists.

Definition 3.9: Absolute soft multiset

A Soft multiset (F, A) over universe U is said to be absolute soft multiset denoted by \tilde{A} if for all $e \in A, F(e) = U$

Definition 3.10: Null soft multiset

A Soft multiset (F, A) over universe U is said to be null soft multiset denoted by $\tilde{\emptyset}$ if for all $e \in A, F(e) = \emptyset$

Definition 3.11: Difference

Let (F, A) and (G, B) be two soft multisets over U , and

$$(G, B) \subseteq (F, A). \text{ Then } M_{(F,A) - (G,B)}(x) = M_{(F,A)}(x) - M_{(F,A)} \cap M_{(G,B)}(x)$$

for all $x \in U$.

It is sometimes referred to as the arithmetic difference of (G, B) from (F, A) . Note that, even if $(G, B) \subset (F, A)$, this definition still holds good

Definition 3.12: Direct Sum of two soft multisets

Let (F, A) and (G, B) be two Soft multisets defined by

$$M_{(F,A) \cup (G,B)}(x) = M_{(F,A)}(x) + M_{(G,B)}(x), \text{ for any } x \in U, \text{ direct sum of two numbers.}$$

That is, an object x occurring a times in (F, A) and b times in (G, B) , occurs $a + b$ times in $(F, A) \cup (G, B)$.

Definition 3.13: OR operation

Let (F, A) and (G, B) be two soft multisets over U . Then (F, A) OR (G, B) denoted by

$(F, A) \vee (G, B)$ is defined as

$(F, A) \vee (G, B) = (H, A \times B)$ where

$$H(a, b) = F(a) \cup G(b) = \max (F(a), G(b)).$$

Definition 3.14: AND operation on two soft multisets

Let (F, A) and (G, B) be two soft multisets over U . Then (F, A) AND (G, B) denoted by

$(F, A) \wedge (G, B)$ is defined as $(F, A) \wedge (G, B) = (H, A \times B)$ where

$$H(a, b) = F(a) \cap G(b) = \min(F(a), G(b)).$$

Proposition 3.2. Let $(F, A), (G, B)$ and (H, C) be three soft multiset over U . Then

- (i) $(F, A) \wedge ((G, B) \wedge (H, C)) = ((F, A) \wedge (G, B)) \wedge (H, C).$
- (ii) $(F, A) \vee ((G, B) \vee (H, C)) = ((F, A) \vee (G, B)) \vee (H, C).$
- (iii) $(F, A) \wedge (F, A) = (F, A).$

Proof (i) By using definition 3.14

$$\begin{aligned} (F, A) \wedge ((G, B) \wedge (H, C)) &= (F, A) \wedge (G, B \times C) = (N, A \times B \times C), \text{ where for} \\ \text{all } (b, c) \in B \times C, M(b, c) &= G(b) \cap H(c) \text{ and for all } (a, b, c) \in A \times B \times C, \\ N(a, b, c) &= F(a) \cap M(b, c) = F(a) \cap (G(b) \cap H(c)) = (F(a) \cap G(b)) \cap \\ H(c) &= Q(a, b) \cap H(c) \text{ with } Q(a, b) = F(a) \cap G(b). \end{aligned}$$

$(Q, A \times B) \wedge (H, C) = ((G, B) \wedge (H, C)) \wedge (H, C)$. Hence (i) has been proved.

(ii) Similar to proof of (i), (ii) can be proved with the help of definition 3.13.

The proof of (iii) is straight forward, hence omitted.

Definition 3.15: Disjoint Soft sets

Let (F, A) and (G, B) be two soft multisets over a common universe U . Then

(F, A) and (G, B) are said to be disjoint if

$(F, A) \cap (G, B) = (H, C)$. Where $C = A \cap B = \emptyset$ and for every $e \in C$,

$$H(e) = F(e) \cap G(e) = \emptyset$$

Definition 3.16: Complement of a soft multiset

The complement of a soft multiset (F, A) is denoted by $\overline{(F, A)}$ and is defined by $\overline{(F, A)} = M_U(x) - M_{(F, A)}(x)$ for all $x \in U$.

$$\overline{(F, A)} = (\bar{F}, \lrcorner A) \text{ where } \bar{F}: \lrcorner A \rightarrow U.$$

Theorem 3.1: De Morgan's Laws;

Let (F, A) and (G, B) be two soft multisets over U , then

- i. $\overline{[(F, A) \vee (G, B)]} = \overline{(F, A)} \wedge \overline{(G, B)}$
- ii. $\overline{[(F, A) \wedge (G, B)]} = \overline{(F, A)} \vee \overline{(G, B)}$

Proof:

- i. $\overline{[(F, A) \vee (G, B)]} = \overline{(F, A)} \wedge \overline{(G, B)}$
 $(F, A) \vee (G, B) = (J, A \times B)$

Then $\overline{[(F, A) \vee (G, B)]} = \overline{(J, A \times B)} = (\bar{J}, \lrcorner(A \times B))$

Now, taking $(\lrcorner a, \lrcorner b) \in \lrcorner(A \times B)$

$$\begin{aligned} \bar{J}(\lrcorner a, \lrcorner b) &= U - J(a, b), \\ &= U - [F(a) \cup G(b)] \\ &= [U - F(a)] \cap [U - G(b)], \\ &= \bar{F}(\lrcorner a) \cap \bar{G}(\lrcorner b) \end{aligned}$$

$$\begin{aligned} \overline{(F, A)} \wedge \overline{(G, B)} &= (\bar{F}, \lrcorner A) \wedge (\bar{G}, \lrcorner B) \\ &= (K, \lrcorner A \times \lrcorner B) \\ &= (K, \lrcorner(A \times B)) \end{aligned}$$

Where $K, (\lrcorner a, \lrcorner b) = \bar{F}(\lrcorner a) \cap \bar{G}(\lrcorner b)$

Since \bar{J} and K are the same, therefore, we conclude that

$$\overline{[(F, A) \vee (G, B)]} = \overline{(F, A)} \wedge \overline{(G, B)}$$

- ii. $\overline{[(F, A) \wedge (G, B)]} = \overline{(F, A)} \vee \overline{(G, B)}$
 $(F, A) \wedge (G, B) = (H, A \times B)$

Then $\overline{[(F, A) \wedge (G, B)]} = \overline{(H, A \times B)} = (\overline{H}, \uparrow(A \times B))$

Now, take $(\uparrow a, \uparrow b) \in \uparrow(A \times B)$.

Therefore, $\overline{H}(\uparrow a, \uparrow b) = U - H(a, b)$,

$$\begin{aligned} &= U - [F(a) \cap G(b)], \\ &= [U - F(a)] \cup [U - G(b)], \\ &= \overline{F}(\uparrow a) \cup \overline{G}(\uparrow b) \end{aligned}$$

Now, $\overline{(F, A)} \vee \overline{(G, B)} = (\overline{F}, \uparrow A) \vee (\overline{G}, \uparrow B)$,

$$\begin{aligned} &= (M, \uparrow A \times \uparrow B) \\ &= (M, \uparrow(A \times B)). \end{aligned}$$

Where $M(\uparrow a, \uparrow b) = \overline{F}(\uparrow a) \cup \overline{G}(\uparrow b)$.

Since \overline{H} and M are the same, hence the Proof.

Proposition 3.2

$$1. \overline{\overline{(F, A)}} = (F, A),$$

Proposition 3.3

If $(F, A), (G, B)$ and (H, C) are three Soft multisets over U , then

1. $(F, A) \cup ((G, B) \cup (H, C)) = ((F, A) \cup (G, B)) \cup (H, C)$,
2. $(F, A) \cup (F, A) = (F, A)$,
3. $(F, A) \cap ((G, B) \cap (H, C)) = ((F, A) \cap (G, B)) \cap (H, C)$
4. $(F, A) \cap (F, A) = (F, A)$.

Proof: The proof is straight forward.

4. Some Properties holding for soft multiset operations

If $(F, A), (G, B)$ and (H, C) are three Soft multisets over U , then the following properties holds:

i. Commutativity:

$$\begin{aligned} (F, A) \downarrow (G, B) &= (G, B) \downarrow (F, A), \\ (F, A) \cup (G, B) &= (G, B) \cup (F, A). \end{aligned}$$

ii. Associativity:

$$(F, A) \Downarrow ((G, B) \Downarrow (H, C)) = ((F, A) \Downarrow (G, B)) \Downarrow (H, C).$$

iii. **Idempotency:**

$$(F, A) \cup (F, A) = (F, A),$$

$$(F, A) \cap (F, A) = (F, A) \text{ but } (F, A) \Downarrow (F, A) \neq (F, A).$$

iv. **Distributivity:**

$$(F, A) \Downarrow ((G, B) \cup (H, C)) = ((F, A) \Downarrow (G, B)) \cup ((F, A) \Downarrow (H, C)),$$

$$(F, A) \Downarrow ((G, B) \cap (H, C)) = ((F, A) \Downarrow (G, B)) \cap ((F, A) \Downarrow (H, C)),$$

$$(F, A) \cup ((G, B) \cap (H, C)) = ((F, A) \cup (G, B)) \cap ((F, A) \cup (H, C)),$$

$$(F, A) \cap ((G, B) \cup (H, C)) = ((F, A) \cap (G, B)) \cup ((F, A) \cap (H, C)).$$

The proof of all these identities follows from the interpretation of \cup , \cap and \Downarrow of two natural numbers as maximum, minimum and (direct) Sum respectively.

It is easy to see that \Downarrow is stronger than both \cup and \cap in the sense that neither \cup nor \cap distributes over \Downarrow , where as \Downarrow distributes over both \cup and \cap .

5. Conclusion

The soft set theory proposed by Molodtsov offers general mathematical tools for dealing with uncertain, fuzzy or vague concepts. In this paper, soft multiset is redefined, which is a mapping from set parameters to power multiset. Moreover, basic set theoretic operations such as union, intersection, AND-product, OR-product and complement are also redefined to capture the notion of multiset. Basic properties of these operations are also investigated and some results proved.

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